A non-linear geodetic data inversion using ABIC for slip distribution on a fault with an unknown dip angle

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SUMMARY
We have developed a method of geodetic data inversion for slip distribution on a fault with an unknown dip angle. A common strategy for obtaining slip distribution in previous studies is to first determine the fault geometry by minimizing the square misfit under the assumption of a uniform slip on a rectangular fault, and then apply the usual linear inversion technique to estimate a slip distribution on the determined fault. It is not guaranteed, however, that the fault determined under the assumption of a uniform slip gives the best fault geometry for a spatially variable slip distribution. The inverse problem is non-linear for cases with unknown fault geometries, but the non-linearity of the problems is actually weak, when we can assume the fault surface to be a flat plane. In particular, when a clear trace of coseismic faults is observed on the Earth’s surface, only the dip angle is an unknown parameter to determine the fault geometry. Then, we regarded the dip angle as an hyperparameter that prescribed the structure of parametric models, and obtained the best estimate of the dip angle using Akaike's Bayesian Information Criterion (ABIC). With the best estimate of the dip angle, we can obtain the slip distribution on the fault based on the maximum-likelihood principle. We applied the method to the InSAR data of the 1995 Dinar, Turkey earthquake and obtained a much lower dip angle than the previous analyses.

Key words: Inverse theory; Satellite geodesy; Earthquake source observations.

1 INTRODUCTION
Large, shallow earthquakes cause significant surface displacements around their focal area. These coseismic surface displacements can be measured by the comparison of pre- and post-seismic geodetic observations. Elastic dislocation theory quantitatively relates surface displacements to a distribution of slip on a fault surface (e.g. Maruyama 1964; Okada 1985; Fukahata & Matsu’ura 2005). It is, therefore, possible to set up the inverse problem of reconstructing the slip distribution on a fault surface from observed geodetic data.

The problems of geodetic data inversion may be divided into two classes according to whether the geometry of a fault surface is known. When the fault geometry is known, which is usually the case for large interplate earthquakes, the problem is basically linear; that is, if we have an enough data with good accuracy, we can obtain the solution of inverse problems by the linear least-squares method based on the maximum-likelihood principle. For most geophysical observations, however, data are inaccurate and insufficient, and so we need to introduce some prior constraints (Jackson 1979) in order to compromise reciprocal requirements for model resolution and estimation errors in a natural way (Backus & Gilbert 1970). For earthquake slip distributions, a smoothness constraint is commonly applied. Usually, such problems have been solved within the framework of the linear inversion method by adjusting the smoothness parameter manually (e.g. Du et al. 1992; Feigl et al. 2002; Kaverina et al. 2002). Strictly speaking, however, the inverse problem with prior constraints is non-linear, because the equation to be solved has a product term of unknown model parameters, which give the slip distribution on a fault surface, and an unknown hyperparameter, which prescribes the relative weight of information from observed data and prior constraints. So, we cannot determine the optimal values of the model parameters and the hyperparameter at the same time with the usual linear inversion method. Yabuki & Matsu’ura (1992) have dealt with this kind of problems to estimate a spatial distribution of coseismic fault slip. In their analysis, the optimal value of the hyperparameter was first determined in an objective way by using a Bayesian Information Criterion (ABIC) proposed by Akaike (1980) on the basis of the entropy maximization principle (Akaike 1977), and then the optimal values of model parameters were determined by the linear maximum-likelihood method. The inversion method developed by Yabuki & Matsu’ura (1992) has been widely applied to geodetic data (e.g. Yoshioka et al. 1993; Fukahata et al. 1996; Sagitaya 1999; Fukahata et al. 2004), to seismic data for source processes (e.g. Yoshida 1989;
When the fault geometry is unknown, which is the case for most intraplate earthquakes, the inverse problem is essentially non-linear. This class of problems has been treated by many researchers (e.g. Matsu’ura 1977a,b; Matsu’ura & Hasegawa 1987; Feigl et al. 1995; Clarke et al. 1997; Wright et al. 1999). A common strategy for obtaining slip distribution is to first determine the fault geometry by minimizing the square misfit under the assumption of a uniform slip on a rectangular fault, and then apply the usual linear inversion technique to estimate a slip distribution on the determined fault (e.g. Árnadóttir & Segall 1994; Jónsson et al. 2002; Wright et al. 2003; Schmidt & Bürgmann 2006; Pathier et al. 2006). It is not guaranteed, however, that the fault determined under the assumption of a uniform slip gives the best fault geometry for a spatially variable slip distribution. In addition, in obtaining a uniform slip fault model, we have to simultaneously determine the values of the nine mutually dependent parameters, such as the strike and the width of the fault, which is a highly non-linear process.

Although the inverse problem is non-linear for cases with unknown fault geometries, the non-linearity of the problems is actually weak (Johnson et al. 2001; Simons et al. 2002), when we can assume the fault surface to be flat. In particular, when a clear fault trace is observed on the Earth’s surface after an earthquake, we can estimate the strike and the location of the fault very precisely. In this case, if we take a sufficiently large fault plane, only the dip angle has large ambiguity. Therefore, extending the formulation developed by Yabuki & Matsu’ura (1992), we can dissolve the non-linearity of the inverse problem by treating the dip angle as another hyperparameter to be determined with ABIC.

In Section 2, we formulate a method of geodetic data inversion to estimate a coseismic slip distribution on a fault with an unknown dip angle. In Section 3, we apply this inversion method to the Synthetic Aperture Radar interferometry (InSAR) data of the 1995 Dinar earthquake, Turkey, and demonstrate its validity.

2 MATHEMATICAL FORMULATION

2.1 Observation equations for InSAR data

In this section, we formulate an inverse problem to estimate coseismic slip distribution on a fault from InSAR data. Here, we assume that the fault surface is expressed by a flat plane and that the fault trace on the Earth’s surface is well determined from observation. Then, the only unknown parameter to determine the fault geometry is the dip angle.

We consider a sufficiently large fault plane with a dip angle \(\delta\) as shown in Fig. 1, where we take the \(x\)-axis to be the fault strike and the \(z\)-axis to be vertical to the Earth’s surface (geoid) with downward positive. The \(y\)-axis is taken to construct the right-hand system and the origin of the coordinate system is set at the centre of the fault trace.

Static coseismic surface displacements \(w_p\) caused by a dislocation \(u_q\) on a fault surface is generally expressed by

\[
w_p(x, y, 0) = \int G_{pq}(x, y, 0; x', y', z') u_q(x', y', z') dS,
\]

where \(S\) represents the fault surface and \(G\) is the Green’s function for an elastic half-space, which has been obtained by many researchers (e.g. Maruyama 1964; Matsu’ura 1977a; Okada 1985). The subscripts \(p\) and \(q\) denote the components (e.g. \(x, y, z\)) of the surface displacement and the dislocation, respectively. Because we assumed the fault surface to be a flat plane with a dip angle \(\delta\), we can rewrite eq. (1) as

\[
w_p(x, y, 0) = \int G_{pq}(x, y, 0; x', z'/\tan \delta, z') u_q(x', z'; \delta) \frac{1}{\sin \delta} dx'dz'.
\]

Figure 1. Setting of the coordinate system and the fault plane. \(\delta\) denotes the dip angle of the fault.
Next, we represent the slip distribution \( u_q \) on the fault by linear combination of a finite number of basis functions:

\[
u_q(x, z; \delta) = \sum_{k=1}^{K} \sum_{l=1}^{L} a_{kl} X_k(x) Z_l(z),\]

where \( a_{kl} \) are the expansion coefficients (model parameters) to be determined from observed data and \( X_k(x) \) and \( Z_l(z) \) are the basis functions for \( x \) and \( z \) on the fault surface. In the following, we consider only the dip-slip component for simplicity, and as for the basis functions we use the normalized cubic B-splines with an equally spaced local support.

To measure coseismic surface displacements, we use InSAR data \( d(x_i, y_i) \), where \( (x_i, y_i) \) denotes the surface position. Since InSAR data give the line-of-sight component of surface displacements, they can be related to the surface displacement \( w_p \) in the following equation:

\[
d(x_i, y_i) = c_1 w_1(x_i, y_i, 0) + c_2 w_2(x_i, y_i, 0) + f + e_i,
\]

where \( c_1 \) and \( c_2 \) are the \( x \) and \( y \) components of the unit vector pointing from the ground to the satellite, respectively. From eqs (2)–(4), we obtain observation equations for InSAR data:

\[
d(x_i, y_i) = \sum_{k=1}^{K} \sum_{l=1}^{L} a_{kl} H_{kl}(\delta) + f + e_i,
\]

where

\[
H_{kl}(\delta) = c_1 \Phi_{k,1l}(\delta) + c_2 \Phi_{k,2l}(\delta) + c_2 \Phi_{z,kl}(\delta)
\]

with

\[
\Phi_{k,jl}(\delta) = \int G_{pq} \left( x_i, y_i, 0; x', z' / \tan \delta, z' \right) X_k(x') Z_l(z') \frac{1}{\sin \delta} dx' dz'.
\]

It is possible in eq. (5) to regard the unknown offset \( f \) as a model parameter to be determined from observed data. So, rearranging the model parameter \( a_{kl} \) including the unknown offset, we define a \( M + 1 \) (= \( K \times L + 1 \)) dimensional model parameter vector \( a \) as

\[
a^T = [a_{11}, a_{12}, \ldots, a_{1L}; a_{21}, a_{22}, \ldots, a_{2L}; \ldots; a_{K1}, a_{K2}, \ldots, a_{KL}; f].
\]

In a similar way, we define a \( N \times (M + 1) \) dimensional coefficient matrix as

\[
H(\delta) = \begin{pmatrix}
H_{11} & H_{12} & \cdots & H_{1K} \\
H_{21} & H_{22} & \cdots & H_{2K} \\
\vdots & \vdots & \ddots & \vdots \\
H_{N1} & H_{N2} & \cdots & H_{NK}
\end{pmatrix}.
\]

where \( N \) denotes the number of data. Using \( a \) and \( H(\delta) \), we can rewrite the observation eq. (5) in a vector form:

\[
d = H(\delta)a + e.
\]

Here, it should be noted that the observation eq. (10) has a product term of the model parameters \( a \) and a function of the unknown dip angle \( \delta \), which represents the non-linearity of the problem. Except for that, eq. (10) has the same form as for the usual linear inverse problems. Hence we can obtain the solution without iteration.

We assume the error \( e \) of the data to be Gaussian, with zero mean and covariance \( \sigma^2E \), whose \( ij \) component is expressed as

\[
E_{ij} = \exp \left( -\frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{s} \right).
\]

where \( s \) represents the characteristic correlation distance of the errors, which largely come from the variations of atmospheric water vapour. We take \( s \) to be 10 km in this analysis (Wright et al. 2003). We cannot neglect the contribution of the cross terms \( E_{ij}(i \neq j) \) of the covariance matrix for InSAR data, because they have strong spatial correlation (Lohman & Simons 2005). Neglecting spatial correlations can cause serious problems, which will be illustrated in Section 3.3. With the covariance matrix \( \sigma^2E \), we have a probability density function (pdf) of observed data \( d \) for given model parameters \( a \) as

\[
p(d|a; \sigma^2, \delta) = (2\pi \sigma^2)^{-N/2} \exp \left\{ -\frac{1}{2\sigma^2} [d - H(\delta)a]^T E^{-1} [d - H(\delta)a] \right\}.
\]

### 2.2 Inversion algorithm using ABIC

It is considered that the slip distribution on seismic faults must be smooth in some degree due to finiteness of the fracture strength of rocks. As a measure of roughness of the slip distribution, according to Yabuki & Matsu’ura (1992), we introduce the following quantity

\[
r(\delta) = \int \int \left[ \frac{\partial^2 u(x, z; \delta)}{\partial x^2} \right]^2 + 2 \sin \delta \frac{\partial^2 u(x, z; \delta)}{\partial x \partial z} \right]^2 + \left[ \sin^2 \delta \frac{\partial^2 u(x, z; \delta)}{\partial z^2} \right]^2 \frac{1}{\sin \delta} dx dz.
\]
Substituting eq. (3) into eq. (13), we obtain

\[
r(\delta) = \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{p=1}^{L} a_{kl} G_{klp}(\delta) a_{pq}
\]

with

\[
G_{klp}(\delta) = \frac{1}{\sin \delta} \left[ \frac{\partial^2 X_4(x)}{\partial x^2} \frac{\partial^2 X_p(x)}{\partial x^2} dx \int Z_l(z) Z_p(z) dz \right. \\
\left. + 2 \sin \delta \int \frac{\partial X_4(x)}{\partial x} \frac{\partial X_p(x)}{\partial x} dx \int \frac{\partial Z_l(z)}{\partial z} \frac{\partial Z_p(z)}{\partial z} dz + \sin^2 \delta \int X_4(x) X_p(x) dx \int \frac{\partial^2 Z_l(z)}{\partial z^2} \frac{\partial^2 Z_p(z)}{\partial z^2} dz \right]
\]

or, in vector form,

\[
r(\delta) = a^T G(\delta) a,
\]

where we put zero for every component of the \((M+1)\)th column and \((M+1)\)th row of the matrix \(G\). Since the quantity \(r\) has a positive-definite quadratic form of \(a\), we may represent the prior constraints on the roughness of the fault slip distribution in the form of a probability density function with a hyperparameter \(\rho^2\) as

\[
p(a; \rho^2, \delta) = (2\pi \rho^2)^{-P/2} \|\Lambda_p(\delta)\|^{1/2} \exp \left[ -\frac{1}{2\rho^2} a^T G(\delta) a \right],
\]

where \(P\) is the rank of the matrix \(G\), and \(\|\Lambda_p(\delta)\|\) represents the absolute value of the product of non-zero eigenvalues of \(G\).

Now we incorporate the prior distribution \(p(a; \rho^2, \delta)\) in eq. (17) with the data distribution in eq. (12) by using Bayes’ theorem, and construct a highly flexible model with the hyperparameters \(\sigma^2\) and \(\rho^2\) and a dip angle \(\delta\):

\[
p(a; \sigma^2, \rho^2, \delta | d) = c p(d | a; \sigma^2, \rho^2, \delta) p(a; \sigma^2, \rho^2, \delta),
\]

where \(c\) is a normalizing factor independent of the model parameters \(a\) and hyperparameters \(\sigma^2\) and \(\rho^2\) and a dip angle \(\delta\). The Bayesian model in eq. (18) consists of a family of parametric models; that is, different values of the hyperparameters and the dip angle give different parametric models. In this meaning we can regard the dip angle \(\delta\) as a hyperparameter, which prescribes the structure of parametric models. The selection of a specific model from among the family of parametric models can be objectively done by using a Bayesian information criterion (ABIC), proposed by Akaike (1980). In the present case, where the number of adjusted hyperparameters \(\sigma^2, \rho^2\) and \(\delta\) is definite, ABIC is given as

\[
\text{ABIC}(\sigma^2, \rho^2, \delta) = -2 \log \int p(a; \sigma^2, \rho^2, \delta | d) da + C.
\]

The values of \(\sigma^2, \rho^2\) and \(\delta\) which minimize ABIC are chosen as the best estimate of them. Carrying out the integration in eq. (19) and changing the hyperparameter from \(\rho^2\) to \(\sigma^2(= \sigma^2/\rho^2)\), we obtain

\[
\text{ABIC}(\sigma^2, \delta) = (N + P - M) \log s(a^*; \sigma^2, \delta) - P \log \sigma^2 - \log \|\Lambda_\rho(\delta)\| + \log \|H(\delta)^T E^{-1} H(\delta) + \sigma^2 G(\delta)\| + C',
\]

where

\[
s(a; \sigma^2, \delta) = [d - H(\delta)a]^T E^{-1} [d - H(\delta)a] + \sigma^2 a^T G(\delta) a
\]

with

\[
a^*(\sigma^2, \delta) = \left[H(\delta)^T E^{-1} H(\delta) + \sigma^2 G(\delta)\right]^{-1} H(\delta)^T E^{-1} d.
\]

Here, we used

\[
\sigma^2 = s(a^*; \sigma^2, \delta) / (N + P - M),
\]

which is derived from the necessary condition that the partial derivative of the ABIC with respect to \(\sigma^2\) must be zero (Akaike 1980).

The search for the values of \(\sigma^2\) and \(\delta\) which minimize ABIC in eq. (20) can be carried out numerically. Once the values of \(\sigma^2\) and \(\delta\) minimizing ABIC has been found, based on the maximum likelihood method, the best estimates of the model parameters \(a^*\) are directly obtained from eq. (22) by substituting those values. Then, the covariance matrix \(C\) of the model parameters \(a^*\) is given as

\[
C = \frac{s(a^*; \sigma^2, \delta)}{N + P - M} \left[H(\delta)^T E^{-1} H(\delta) + \sigma^2 G(\delta)\right]^{-1},
\]

and the resolution matrix \(R\) is

\[
R = \left[H(\delta)^T E^{-1} H(\delta) + \sigma^2 G(\delta)\right]^{-1} H(\delta)^T E^{-1} H(\delta).
\]

### 3. APPLICATION TO InSAR DATA OF DINAR EARTHQUAKE

#### 3.1 Observed data

Southwest Anatolia (Turkey) is highly seismically active and forms part of the Aegean domain of distributed N–S extension (Taymaz et al. 1991; Jackson 1994; Fig. 2). Global Positioning System measurements indicate a regional extension rate of 10–15 mm yr\(^{-1}\) (Reilinger et al.
The 1995 October 1, $M_s = 6.1$, Dinar earthquake ruptured a section of the Dinar–Çivil fault. The fault is characterized by a 60 km scarp with up to 1500 m of relief to the NE, although there is only approximately half this relief at the rupture location. The earthquake created a 10 km continuous surface rupture running along the base of the scarp with a maximum vertical offset of 25–30 cm, tailing off to around 15 cm to the SE (Eyidogan & Barka 1996). The topography has a classic tilted block geometry with a steep scarp against the fault plane and gently tilted backslope.

Coseismic displacements from the Dinar earthquake are measured using ERS SAR images spanning the event (Fig. 3). We processed the data with the ROI pac software at JPL, using a DEM constructed from an ERS tandem pair to correct for topographic phase and precise orbits supplied by Delft University to apply baseline corrections. The corrected coseismic interferogram shows 21 fringes in the hanging wall of the fault indicating a maximum line-of-sight downthrown displacement of 0.59 m. The hanging-wall fringe pattern indicates asymmetrical deformation with the maximum change in range towards the NW end of the observed surface rupture but about 2 km away from it. Three upthrown fringes (85 mm) appear in the footwall of the fault after the topographic correction.

The InSAR data had already been inverted to estimate the source parameters of the Dinar earthquake by Wright et al. (1999). They considered the models of one- and four-segment faults with a uniform slip on each model. The estimated optimal dip angles were 53.8° and 49° for the one- and four-segment models, respectively. Using seismic data, several researchers (Eyidogan and Barka 1996; Pinar 1998; Wright et al. 1999; Utkucu et al. 2002) have also estimated the source parameters. Due to the difference of the data sets and the inversion methods, the obtained dip angle has a large variety. For example, Harvard CMT solution gave 30°. Eyidogan and Barka (1996) and Pinar (1998), who considered the earthquake was composed of two subevents with different fault geometries, obtained 40° and 34° for the first smaller event and 62° and 40° for the second larger event, respectively. Wright et al. (1999), who used SH-wave data as well as P-wave data, estimated the dip angle to be 43°. Utkucu et al. (2002) determined a slip distribution of the earthquake, using the fault geometry obtained by Pinar (1998). The estimated rakes, on the other hand, coherently show the earthquake had a nearly pure dip slip, except for the second subevent by Eyidogan and Barka (1996).

3.2 Inversion results

We subsample the InSAR data using Quadtree algorithm (Jonsson et al. 2002). In this scheme, the unwrapped interferograms are divided into quadrants. If the variance with each quadrant exceeds a specific threshold, the quadrant is subdivided into four. This process is repeated until the variance within the quadrant is less than the threshold. We used a maximum block size of 5 km, which corresponds to the half of a typical correlation length scale for atmospheric noise. Furthermore, in order to maintain the resolution of fault slip, we put data points with
Figure 3. Interferogram for the Dinar earthquake showing the location of the surface rupture (white line) together with the model fault trace for the inversion analysis (thick red line). The interferogram is formed from radar scenes acquired by ERS-1 on 1995 August 13 and ERS-2 on 1996 January 01 (Standard descending track 293). It has a perpendicular baseline of 8 m, and any residual topographic phase has been corrected using a DEM derived from an ERS tandem pair (1995 October 22,23).

Figure 4. A diagram showing the basis functions to represent the slip distribution on the fault plane in the inversion analysis.

The interval of 1 km near the fault on the footwall, where the displacements were quite flat. The total number of data is 898. In inverting the InSAR data we use an infinite elastic half-space model with Lamé constants $\lambda = 3.22 \times 10^{10}$ Pa and $\mu = 3.43 \times 10^{10}$ Pa.

We take an adequately long fault plane (24 km) passing through the surface rupture (Fig. 3), where the maximum depth of the fault plane is taken to be 12 km and the dip angle can be taken arbitrarily. The strike of the model fault is 145°. As shown in Fig. 4, the interval of the knots for basis functions of the cubic B-splines, $X_k(x)$ and $Z_l(z)$, is taken to be 2 km both for the direction of depth and along the strike of the fault. Then, the number of basis functions is 9 for $x$ and 5 for $z$ ($K = 9$ and $L = 5$). In experiments where we took a larger fault and/or a smaller interval of knots, almost the same inverted slip distributions were obtained. As mentioned in the preceding section, we consider only the dip-slip component, because the previous inversion analyses of seismic and InSAR data show the Dinar earthquake to be a nearly pure dip slip and InSAR data give only 1-D (line-of-sight) displacements.

We compute the values of ABIC ($\alpha^2$, $\delta$) from eq. (20), and plot them in a contour map (Fig. 5). The contour, which differs by 2 from the ABIC minimum, is also shown by the broken line. The point of ABIC minimum, shown by the star, gives the best estimates for the
hyperparameters $\alpha^2$ and the dip angle $\delta$. The optimum dip angle is found to be $34^\circ$. Because the difference of one free hyperparameter corresponds to 2 in the value of ABIC (Akaike 1980), a difference of 2 in ABIC has statistical significance. So, we may regard the estimation error of the dip angle as about $\pm 2^\circ$.

With the best estimates of $\alpha^2$ and $\delta$, the optimal values of the model parameters $a$ is obtained from eq. (22). Then, using eq. (3), we can reconstruct the slip distribution $u(x, z; \delta)$ on the fault as shown in Fig. 6(a), where the slip distribution is projected onto a vertical plane. In Fig. 6 the standard deviation (b) and resolution (c) of the slip are also shown. An area of 6 km length and 3 km depth has a slip more than 1 m with the maximum slip of about 1.3 m. The calculated seismic moment $M_0$ is $4.1 \pm 0.2 \times 10^{18}$ N m. These features are reasonably consistent with the result of Wright et al. (1999). In particular, a relatively high slip area in the deeper northwest of the fault is recognized, as strongly suggested by their study.

In the previous inversion analysis of InSAR data (Wright et al. 1999), as mentioned above, the dip angle of about $50^\circ$ has been obtained under the assumption of a uniform slip on rectangular faults. So, for reference, we invert the InSAR data fixing the dip angle at $50^\circ$. The best estimate of the hyperparameter $\alpha^2$ is determined by ABIC minimum even in this case. The obtained slip distribution (Fig. 7a) is much rougher than that for the optimum model with the dip angle of $34^\circ$. The standard deviation (Fig. 7b) is also twice as large as for the optimal model, while the resolution (Fig. 7c) is similar. In addition, there are some portions of negative slip larger than 0.4 m. If we adopt a non-negative least-squares method (Lawson & Hanson 1974), it is possible to eliminate such negative slip areas. However, we have to recall that we originally assumed a Gaussian noise for observed data, which gave the basis of the least-squares criterion. From eq. (10) and the law of propagation of errors, the errors of model parameters $a$ also follow the Gaussian distribution. The non-negative method contradicts this condition. Therefore, if possible, it is preferable not to impose the non-negative constraint to maintain the consistency of the inversion analysis.

Given the slip distribution (Fig. 6a), we can calculate line-of-sight crustal displacements (Fig. 8a). By subtracting the model interferogram (Fig. 8a) from the observed data (Fig. 3), the residual interferogram is obtained (Fig. 8b). The residual is very small and generally less than 1 interference fringe.

### 3.3 Importance of covariance

Before the introduction of ABIC, we have not had a definitive way to determine the relative weight between the information from observed data and prior constraints. The point of ABIC is that we can objectively determine it based on statistics. That is to say, when we have an enough amount of accurate data, a model that well explains observed data is selected. Conversely, when the data are inaccurate and/or insufficient, the model comes to follow prior constraints. This is why, in the inversion analysis with ABIC, we have to be more careful in dealing with information included in observed data.

InSAR gives us spatially dense crustal displacement data, if the coherence is good. Here, we encounter the problem of sampling interval. For example, if we pick up InSAR data very densely, the information from observed data apparently increases. So, if we invert such a data set, a model that excessively fits observed data would be selected.
Figure 6. Slip distribution (a) of the Dinar earthquake projected onto a vertical plane. The standard deviation (b) and resolution (c) of the slip are also shown. The optimal dip angle (34°) is used.

Figure 7. Slip distribution (a) of the Dinar earthquake projected onto a vertical plane. The standard deviation (b) and resolution (c) of the slip are also shown. The reference dip angle (50°) is used.
As mentioned above, InSAR data include errors that are highly spatially correlated, mainly resulting from changes in atmospheric conditions. In other words, each data is not completely independent. So, taking the effect of data covariance into account, we can reasonably reduce the information from densely sampled observed data, which enables us to avoid a biased inversion.

In Fig. 9 we show the slip distribution on the fault inverted from the same InSAR data with the optimal dip 34° (a) and the reference dip 50° (b), neglecting the off-diagonal components of the covariance matrix. The best estimate of the hyperparameter $\alpha^2$ is again determined by ABIC. As shown in Fig. 9, the obtained slip distributions are rougher than solutions where covariance is considered, particularly for the case of the reference dip 50°, where the obtained slip distribution is unnaturally rough. When we neglect the effect of covariance, the apparent information contained in the observed data becomes too high. If we can manually adjust the value of hyperparameter $\alpha^2$, which prescribe the relative weight between the information from observed data and prior constraints, the effect of neglecting the covariance is limited. When we use ABIC, however, it autonomously determine the relative weight. Then, the effect of neglecting the covariance can be crucial—resulting in overly rough solutions. Our results (Fig. 9) suggest that this effect is particularly strong if the adopted model is not close to the optimal one.

4. DISCUSSION AND CONCLUSIONS

We have developed a method of geodetic data inversion for slip distribution on a fault with an unknown dip angle. The method is much simpler than the previous one, in which a uniform slip on a rectangular fault has been assumed in determining the fault geometry. The result
Figure 9. Inverted slip distribution on the fault with the optimal dip 34° (a) and the reference dip 50° (b), obtained by neglecting the off-diagonal components of the covariance matrix. Note that the contour interval is 0.2 m for (a) while 0.4 m for (b).

obtained by this method must be more reasonable, because a wider parametric area, including the case of a particular dip angle obtained by the previous method, is searched. An important point in the inversion analysis is that we construct a model as generally as possible, which is well realized in this method by apparently regarding the dip angle as an hyperparameter to be determined from ABIC. Fukahata et al. (2003, 2004) extended an inversion method based on ABIC for the case with more than one sort of prior constraints. This study is another extension of the inversion method based on ABIC.

The estimated optimal dip angle (34°) by this study was significantly lower than the one obtained by the previous study (Wright et al., 1999). In general, a uniform slip on a fault causes singularity at the edges of the fault. By dividing the fault into many subfaults, this singularity can be reduced. Furthermore, by using smooth basis functions like the cubic B-spline instead of the box-type function, we can avoid this singularity problem. The higher dip angle in the previous study may be due to this effect. This is consistent with the result of Wright et al. (1999) who obtained a lower dip angle (49°) from their four-segment model than from their one-segment model (53.8°).

It is possible to obtain the fault geometry of earthquakes from inversion analyses of seismic data. A joint inversion of InSAR and seismic data based on ABIC (Funning 2005; Funning et al. 2005), however, clearly shows that teleseismic data has much less information than InSAR data in determining the total slip distributions of earthquakes. Therefore, the fault geometry should also be estimated from InSAR data to maintain the accuracy of the inversion analysis.

The developed method is broadly applicable for estimating slip distributions for many continental earthquakes. In this paper, we focused on the case in which a clear trace of a seismic fault was observed. It would be straightforward to extend the method for the case of general flat
faults. In this case additional two parameters, the strike and location, are needed to determine the fault geometry. The best estimates of these values could also be obtained by the criterion of ABIC minimum. This is a subject of a subsequent paper.

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References


