General expressions for internal deformation fields due to a dislocation source in a multilayered elastic half-space

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SUMMARY
We have obtained general expressions for internal displacement and stress fields due to a point dislocation source in a multilayered elastic half-space under gravity. Most previous expressions for the internal deformation fields were obtained by applying one of two different types of Thomson–Haskell propagator matrix, namely the up-going propagator matrix proposed by Singh (1970) and the down-going propagator matrix proposed by Sato (1971). The solution derived with the up-going propagator matrix is stable below the source, but becomes unstable above the source. In contrast, the solution derived with the down-going propagator matrix is stable above the source, but becomes unstable below the source. We succeeded in unifying the up-going and the down-going propagator matrices into a generalized propagator matrix, and applied it to obtain general expressions that are stable at any depth. By integrating the effects of point sources distributed along an infinitely long horizontal line, we also obtained general expressions for a line dislocation source. We give some examples of internal displacement fields computed with these expressions to examine the effects of layering. Applying the correspondence principle of linear viscoelasticity to the derived elastic solutions, we can obtain the internal viscoelastic displacement and stress fields due to dislocation sources.

Key words: crustal deformation, dislocation, internal deformation, layered media, propagator matrix.

1 INTRODUCTION
In most analyses of crustal deformation, an elastic half-space model is used for simplicity. The elastic half-space may be a reasonable assumption as far as our concerns are limited to short-term, regional crustal deformation such as coseismic deformation. For long-term crustal deformation such as interseismic deformation and postglacial rebound, however, the elastic half-space model is no longer acceptable, because the effects of viscoelastic stress relaxation in the asthenosphere cannot be neglected (Thatcher & Rundle 1984; Matsu’ura & Sato 1989; Fukahata et al. 2004). In general, the viscoelastic solution can be obtained from the associated elastic solution by applying the correspondence principle of linear viscoelasticity (Lee 1955; Radok 1957), and so we need to obtain the elastic solution for a layered earth model first.

The purpose of the present study is to obtain the general expressions for internal deformation fields due to a dislocation source in a multilayered elastic half-space. In this problem, the boundary conditions to be satisfied are stress-free at the surface of the elastic half-space, the continuity of displacement and stress components at each layer interface, a certain amount of tangential displacement discontinuity at a dislocation surface, and the finiteness of displacement and stress components in the depths of the elastic half-space. Such a boundary-value problem can be reduced to a set of linear equations written in a matrix form by using the propagator matrix method developed by Thomson (1950) and Haskell (1953). Historically, this matrix equation has been solved in two different ways: one way is to propagate displacement and stress components from the substratum to the free surface with an up-going propagator matrix (Singh 1970), and the other way is to propagate displacement and stress components from the free surface to the substratum with a down-going propagator matrix (Sato 1971). The solution derived with the up-going propagator matrix is stable below the source, but becomes unstable above the source. In contrast, the solution derived with the down-going propagator matrix is stable above the source, but becomes unstable below the source. In this paper we show that the combination of these two methods gives a numerically stable solution over the whole region.

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Since Steketee (1958) introduced elastic dislocation theory into seismology, many theoretical studies have been undertaken to obtain mathematical expressions for the surface and/or internal deformation fields due to a dislocation source in a layered elastic half-space. For example, Singh (1970) obtained the expression for the surface displacement field by applying the Thomson–Haskell propagator matrix method to the general source representation by Ben-Menahem & Singh (1968). Singh’s solution was, however, numerically unstable, because he used the up-going propagator matrix. Jovanovich et al. (1974a,b) proposed an approximation method to suppress the numerical instability and showed some examples of computed surface displacement and strain fields. Rundle (1978, 1980, 1981, 1982) developed another elaborate method to compute surface deformation fields and extended Singh’s elastic solution to a viscoelastic-gravitational case. Roth (1990) and Ma & Kusznir (1992, 1994) also extended the formulation of Singh’s surface displacement to obtain expressions for internal deformation fields. Here, it should be noted that all these expressions derived with the up-going propagator matrix become numerically unstable at the surface and any depth above the source.

Sato (1971), on the other hand, used the down-going propagator matrix to obtain expressions for surface displacement fields due to a dislocation source in a layered elastic half-space. His solution at the surface was certainly stable, because the down-going propagator matrix gives a stable solution above the source. Following Sato’s formulation, Sato & Matsu’ura (1973) and Matsu’ura & Sato (1975) succeeded in computing the surface displacement and strain fields, respectively, without numerical instability. Some of the approximation techniques used in their computation were not for suppressing numerical instability, but for saving computation time. Matsu’ura et al. (1981) obtained mathematical expressions for viscoelastic surface displacement fields by applying the correspondence principle to Sato’s elastic solution, and then Matsu’ura & Sato (1989) introduced gravitational effects into the mathematical expressions. With the down-going propagator matrix we can derive another expression for the internal deformation fields (Matsu’ura & Sato 1997). This type of expression is stable at the surface and any depth above the source, but becomes numerically unstable at any depth below the source. In order to avoid the numerical instability below the source we must use the up-going propagator matrix instead of the down-going propagator matrix.

Recently, Pan (1997) found the cause of numerical instability and derived a solution that is stable below the source for a layered transversely isotropic medium. Wang (1999) and Wang et al. (2003) derived a stable solution for the internal deformation field, but the reconstruction of vector bases, which is the key in their method to obtaining the stable solution, is unnecessary. With a generalized reflection and transmission coefficient matrix method (Kennett 1983), He et al. (2003) derived expressions for displacement fields due to a tensile and a shear dislocation source, but their method includes some numerical problems in wavenumber integration even in the case of surface displacements.

In Section 2, we introduce a generalized propagator matrix that unifies the up-going and the down-going propagator matrices, and derive general expressions for internal displacement and stress fields due to a dislocation source in a multilayered elastic half-space. For comparison with our previous studies, we use a similar notation to that of Sato (1971) and Matsu’ura et al. (1981). In Section 3, we give some numerical examples of internal displacement fields to examine the effects of layering.

2 MATHEMATICAL FORMULATION

We consider \( n - 1 \) parallel, homogeneous and isotropic elastic layers overlying a homogeneous and isotropic elastic half-space. Every layer and interface is numbered in ascending order from the free surface as shown in Fig. 1(a). The \( j \)th layer is bounded by the \((j - 1)\)th and \( j \)th interfaces. The depth of the \( j \)th interface is denoted by \( H_j \), and the thickness of the \( j \)th layer by \( h_j = H_j - H_{j-1} \). In order to deal with deformation fields in a multilayered medium, it is convenient to take a cylindrical coordinate system \((r, \varphi, z)\) as shown in Fig. 1(b). Here, we take the z-axis to be perpendicular to the layer interfaces and directed into the medium. A point dislocation source is located at \((0,0,d)\) in the \( m \)th layer \((1 \leq m \leq n)\) with a dip angle \( \theta \) and a slip angle \( \chi \).

The static displacement field for each layer can be obtained by solving the following equilibrium equation under appropriate boundary conditions:

\[
(\lambda_j + 2\mu_j)\nabla \cdot \mathbf{u}(r, \varphi, z; j) - \mu_j \nabla \times (\nabla \times \mathbf{u}(r, \varphi, z; j)) + \mathbf{X}(r, \varphi, z; j) = 0, \tag{1}
\]

where \( \mathbf{u} \) represents a displacement vector, \( \lambda_j \) and \( \mu_j \) are the Lamé elastic constants in the \( j \)th layer, and \( \mathbf{X} \) denotes a body force vector that corresponds to a dislocation source in the present problem. Since the source term \( \mathbf{X} \) vanishes in the layer without a source \((j \neq m)\), the displacement field is formally expressed as

\[
\mathbf{u}(r, \varphi, z; j) = \mathbf{u}^s(r, \varphi, z; j) + \delta_{nm} \mathbf{u}^g(r, \varphi, z; m), \tag{2}
\]

where \( \mathbf{u}^s \) and \( \mathbf{u}^g \) represent a particular solution and the general solution of eq. (1), respectively. Hereafter we use the superscripts ‘s’ and ‘g’ to denote the variables related to the particular solution and the general solution, respectively.

2.1 Particular solution for a dislocation source in cylindrical coordinates

According to Takeuchi (1959), a particular solution of eq. (1) with a source at a depth \( d \) in the \( m \)th layer can be written with displacement potential functions, \( \Phi_{1m} \), \( \Phi_{2m} \), and \( \Psi_m \), as

\[
\mathbf{u}^s(r, \varphi, z; m) = \nabla \Phi_{1m}^s - \gamma_m(z - d)\nabla \Phi_{2m}^s + \begin{pmatrix} 0 \\ 0 \\ (2 - \gamma_m)\Phi_{2m}^s \end{pmatrix} + \nabla \times \begin{pmatrix} 0 \\ 0 \\ \Psi_m^s \end{pmatrix}, \tag{3}
\]
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with

\[ \gamma_m = \frac{\lambda_m + \mu_m}{2\lambda_m + 2\mu_m}. \]

It should be noted that the potential functions satisfy Laplace’s equation except for \( z = d \). The \( P-SV \)-type deformation field due to \( \Phi_{1m} \) and \( \Phi_{2m} \) and the \( SH \)-type deformation field due to \( \Psi_{1m} \) are independent of each other, and so we can treat them separately.

Sato (1971) has obtained explicit expressions for the displacement potential functions for a dislocation source in the cylindrical coordinate system. Using Sato’s potential functions, we can write the particular solutions of displacement and stress components in the form of semi-infinite wavenumber integrals:

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The general solution of the homogeneous equation associated with eq. (1) can be written in the same form as eq. (3), but with different $g(j, \varphi, \psi)$:

$$
\mathbf{J}(r, \varphi, \psi) = \begin{pmatrix} a_0(\varphi) J_0(\xi r) \\ a_1(\varphi) J_1(\xi r) \\ a_2(\varphi) J_2(\xi r) \end{pmatrix}, \quad \mathbf{J}'(r, \varphi, \psi) = \frac{\partial}{\partial \varphi} \begin{pmatrix} a_0(\varphi) J_0(\xi r) \\ a_1(\varphi) J_1(\xi r) \\ a_2(\varphi) J_2(\xi r) \end{pmatrix}.
$$

with

$$
a_0(\varphi) = \frac{1}{2} \sin \chi \sin 2\theta, \quad a_1(\varphi) = -\sin \chi \cos 2\theta \sin \varphi + \cos \chi \cos \theta \cos \varphi, \quad a_2(\varphi) = \frac{1}{2} \sin \chi \sin 2\theta \cos 2\varphi + \frac{1}{2} \cos \chi \sin \theta \sin 2\varphi,
$$

and where $\partial_i (i = r, \varphi, z)$ represents partial differentiation with respect to $i$. $J_k(\xi r)$ denotes the Bessel function of order $k$, $\xi$ is the wavenumber, and $\Delta \tau$ is the amount of tangential displacement discontinuity (dislocation) over a unit area. Here, we define the deformation matrices for particular solutions, $\mathbf{Y}^r$ and $\mathbf{Y}^\psi$, by

$$
\mathbf{Y}^r(z; m) = \begin{pmatrix} Y_1^r(z; m) \\ Y_2^r(z; m) \\ Y_3^r(z; m) \end{pmatrix}, \quad \mathbf{Y}^\psi(z; m) = \begin{pmatrix} Y_1^\psi(z; m) \\ Y_2^\psi(z; m) \end{pmatrix}.
$$

Hereafter we omit the $\xi$-dependence of variables for simplicity. The actual elements of the deformation matrices are as follows.

$$
\mathbf{Y}^r(z; m) = e^{-i\xi z} \begin{pmatrix} 2 & -\text{sgn}(z) & 2 \\ 4\text{sgn}(z) & -1 & 0 \\ \text{sgn}(z) & 0 & -\text{sgn}(z) \end{pmatrix} + \gamma_m \begin{pmatrix} 1 - 3|z|\xi & z\xi & -1 - |z|\xi \\ -4\text{sgn}(z) + 3z\xi & 1 - |z|\xi & z\xi \\ -4\text{sgn}(z) + 3z\xi & 1 - |z|\xi & z\xi \end{pmatrix},
$$

$$
\mathbf{Y}^\psi(z; m) = e^{-i\xi z} \begin{pmatrix} \text{sgn}(z) & -1 \\ -1 & \text{sgn}(z) \end{pmatrix},
$$

where $\text{sgn}(z)$ is the sign function, which takes the value of 1 for $z > 0$ and -1 for $z < 0$.

### 2.2 General solution of the homogeneous equation

The general solution of the homogeneous equation associated with eq. (1) can be written in the same form as eq. (3), but with different displacement potential functions, $\Phi_j^x$, $\Phi_j^y$, and $\Psi_j^z$:

$$
\mathbf{u}^x(r, \varphi, z; j) = \nabla \Phi_j^x - \gamma_j z \nabla \Phi_j^x + \begin{pmatrix} 0 \\ 0 \\ (2 - \gamma_j) \Phi_j^x \end{pmatrix} + \nabla \times \begin{pmatrix} 0 \\ 0 \\ \Psi_j^x \end{pmatrix}.
$$

The displacement potential functions in eq. (11) are generally expressed by the superposition of all the mode solutions of Laplace’s equation. In the present problem, however, we need only the mode solutions of the same order as in the particular solution to satisfy the boundary conditions imposed on each interface and the free surface. Thus, following Sato (1971), we may write the displacement potential functions in the following form without loss of generality:

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\[
\begin{align*}
\Phi_i^j &= \frac{3n}{4t} \int_0^\infty \chi_1^j(z; \xi) J(r, \varphi; \xi) d\xi \\
\Phi_j^i &= \frac{3n}{4t} \int_0^\infty \chi_2^i(z; \xi) J(r, \varphi; \xi) d\xi \\
\Psi_j^i &= \frac{3n}{4t} \int_0^\infty \chi_3^i(z; \xi) J(r, \varphi; \xi) d\xi
\end{align*}
\]

with

\[
\begin{align*}
\chi_1^j(z; \xi) &= (A_0^j(\xi)e^{z\xi} + A_{1j}^j(\xi)e^{-z\xi}) \\
\chi_2^j(z; \xi) &= (B_0^j(\xi)e^{z\xi} + B_{1j}^j(\xi)e^{-z\xi}) \\
\chi_3^j(z; \xi) &= (C_0^j(\xi)e^{z\xi} + C_{1j}^j(\xi)e^{-z\xi})
\end{align*}
\]

where \(A_{1j}^j, B_{1j}^j, C_{1j}^j\) and \(C_{1j}^j\) \((k = 0, 1, 2)\) are the potential coefficients of the \(j\)th layer to be determined from the boundary conditions. From eqs (11)–(13) we can obtain the integral expressions for the general solutions of the displacement and stress components as

\[
\begin{align*}
u^i(r, \varphi, z; j) &= \frac{3n}{4t} \left[ \int_0^\infty Y_1^0(z; \xi; j) \partial_j J(r, \varphi; \xi) d\xi + \int_0^\infty Y_1^j(z; \xi; j) \frac{1}{2} \partial_j J(r, \varphi; \xi) d\xi \right] \\
u^j(r, \varphi, z; j) &= \frac{3n}{4t} \left[ \int_0^\infty Y_2^0(z; \xi; j) \partial_j J(r, \varphi; \xi) d\xi - \int_0^\infty Y_2^j(z; \xi; j) \partial_j J(r, \varphi; \xi) d\xi \right] \\
u^k(r, \varphi, z; j) &= \frac{3n}{4t} \int_0^\infty Y_3^k(z; \xi; j) J(r, \varphi; \xi) d\xi
\end{align*}
\]

Then, we have

\[
\begin{align*}
\Phi_i^j &= E_j(z) A_j, \quad Y^j(z; j) = E_j' (z) A_j' \end{align*}
\]

where

\[
A_j = \begin{pmatrix}
A_{0j}^+ + A_{0j}^- & A_{1j}^+ + A_{1j}^- & A_{2j}^+ + A_{2j}^- \\
A_{0j}^+ + B_{0j}^j & A_{1j}^+ + B_{1j}^j & A_{2j}^+ + B_{2j}^j \\
A_{0j}^+ + B_{0j}^j & A_{1j}^+ + B_{1j}^j & A_{2j}^+ + B_{2j}^j
\end{pmatrix}, \quad A_j' = \begin{pmatrix}
C_{1j}^+ + C_{1j}^- & C_{2j}^+ + C_{2j}^- \\
C_{1j}^+ + C_{1j}^- & C_{2j}^+ + C_{2j}^- \\
C_{1j}^+ + C_{1j}^- & C_{2j}^+ + C_{2j}^- \end{pmatrix}
\]

\[
E_j(z) = \begin{pmatrix}
C(z) & 0 & S(z) & 0 \\
S(z) & 2C(z) & C(z) & 2S(z) \\
S(z) & C(z) & C(z) & S(z) \\
C(z) & S(z) & S(z) & C(z)
\end{pmatrix} + \gamma_j 
\]

\[
E_j'(z) = \begin{pmatrix}
C(z) & S(z) \\
S(z) & C(z)
\end{pmatrix}
\]

\[
\begin{align*}
C(z) &= \cosh z \xi, \quad S(z) &= \sinh z \xi.
\end{align*}
\]

Here, \(Y_1^j\) and \(Y_2^j\) are, respectively, the \(k\)th rows of the deformation matrices, \(Y^j\) and \(Y^{j'}\), for general solutions.

#### 2.3 Generalized propagator matrix

As shown in eqs (5)–(6) and (14)–(15), we can see that the problem of computing the displacement and stress components at a given depth is practically reduced to the problem of determining the values of the deformation matrices at that depth. In this subsection, we first define the generalized propagator matrices, which relate the displacement and stress components at a given depth to those at any other depth. Then, we demonstrate how the deformation matrices are continuously propagated from the top to the bottom or from the bottom to the top by using the generalized propagator matrices.

First, we consider the case of a layer without any source \((j \neq m)\). As shown in eq. (16), the deformation matrices \(Y^j\) and \(Y^{j'}\) have the depth-independent potential coefficients \(A_j\) and \(A_j'\) in each layer. Therefore, we can relate the deformation matrices at two different depths, \(z_1\) and \(z_2\), to each other by using the propagator matrices, \(F_j\) and \(F_j'\), as

\[
\begin{align*}
Y_j(z_1) &= \Phi_j(z_1), \quad Y_j'(z_2) = \Phi_j'(z_2) \\
Y_j(z_1) &= \Phi_j(z_1), \quad Y_j'(z_2) = \Phi_j'(z_2)
\end{align*}
\]
\[ Y^d(z; j) = F_j(z_2 - z_1) Y^d(z_1; j), \quad Y^\gamma(z; j) = F'_j(z_2 - z_1) Y^\gamma(z_1; j) \]  
with
\[ F_j(z_2 - z_1) = E_j(z_2) E_j^{-1}(z_1), \quad F'_j(z_2 - z_1) = E'_j(z_2) E'_j^{-1}(z_1). \]  

Here, the superscript \(-1\) indicates the inverse of the corresponding matrix. It should be noted that the propagator matrices \(F_j(z_1 - z_2)\) and \(F'_j(z_1 - z_2)\) include the exponentially increasing factor \(\exp((z_1 - z_2)\xi)\), which causes numerical instability if the problem is not treated properly.

Now we define the generalized propagator matrices as functions of \(z\) by replacing \(z_1\) and \(z_2\) in eq. (21) with 0 and the variable \(z\), respectively, as
\[
\begin{align*}
F_j(z) &= \begin{pmatrix}
C(z) & -S(z) & 2S(z) & 0 \\
-S(z) & C(z) & 0 & 2S(z) \\
0 & 0 & C(z) & S(z) \\
0 & 0 & S(z) & C(z)
\end{pmatrix} + \gamma_j \begin{pmatrix}
z\xi S(z) & S(z) - z\xi C(z) & -S(z) + z\xi C(z) & -z\xi S(z) \\
S(z) + z\xi C(z) & -z\xi S(z) & z\xi S(z) & -S(z) - z\xi C(z) \\
z\xi S(z) & S(z) - z\xi C(z) & -S(z) + z\xi C(z) & -z\xi S(z)
\end{pmatrix}, \\
F'_j(z) &= \begin{pmatrix}
C(z) & S(z) \\
S(z) & C(z)
\end{pmatrix}.
\end{align*}
\]

To derive the above expressions, we used
\[ E_j^{-1}(0) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 + \gamma_j & 2 - \gamma_j & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}. \]  

The generalized propagator matrices defined in eq. (22) have the following properties:
\[
\begin{align*}
F_j(0) &= I, \quad F'_j(0) = I, \\
F_j(z_1 + z_2) &= F_j(z_1) F_j(z_2), \quad F'_j(z_1 + z_2) = F'_j(z_1) F'_j(z_2), \\
F_j(-z) &= F_j(z), \quad F'_j(-z) = F'_j(z),
\end{align*}
\]
where \(I\) and \(I\) are the \(4 \times 4\) and \(2 \times 2\) unit matrices, respectively.

In the case of the \(m\)th layer with a source, we have to take the deformation field generated by the source (i.e. the particular solution) into account. As mentioned in Subsection 2.1, the potential functions of the particular solution also satisfy Laplace’s equation except for \(z = d\). Hence, the relations of the deformation matrices in eq. (20) are still valid for the particular solution for both \(z_1, z_2 > d\) and \(z_1, z_2 < d\):
\[
Y^d(z_2 - d; m) = F_m(z_2 - z_1) Y^d(z_1 - d; m), \quad Y^\gamma(z_2 - d; m) = F'_m(z_2 - z_1) Y^\gamma(z_1 - d; m).
\]

At the source depth \(d\) the deformation matrices \(Y^d\) and \(Y^\gamma\) have certain offsets:
\[
Y^d(0--; m) - Y^d(0++; m) = \Delta_m, \quad Y^\gamma(0--; m) - Y^\gamma(0++; m) = \Delta'_m.
\]

From eq. (10) we can obtain the expressions for \(\Delta_m\) and \(\Delta'_m\) as
\[
\begin{align*}
\Delta_m &= 2 \begin{pmatrix} 0 & 1 & 0 \\ -4 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + \gamma_m \begin{pmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\Delta'_m &= -2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.
\end{align*}
\]

As noted for eq. (2), the total deformation field in the source layer is given by the sum of the general solution and the particular solution, and so we define the total deformation matrices \(Y\) and \(Y^\gamma\) by
\[
\begin{align*}
Y(z; j) &= Y^d(z; j) + \delta_{mj} Y^d(z - d; m), \quad Y^\gamma(z; j) = Y^\gamma(z; j) + \delta_{mj} Y^\gamma(z - d; m).
\end{align*}
\]

Substituting eqs (20), (25) and (26) into eq. (28), we obtain the following relations for the total deformation matrices:
\[
\begin{align*}
Y(z; j) &= F_j(z_2 - z_1) Y(z_1; j) \quad (z_1, z_2 > d \text{ or } z_1, z_2 < d) \\
Y^\gamma(z; j) &= F'_j(z_2 - z_1) Y^\gamma(z_1; j) \\
Y(z; m) &= F_m(z_2 - z_1) Y(z_1; m) - \text{sgn}(z_2 - d) F_m(z_2 - d) \Delta_m \\
Y^\gamma(z; m) &= F'_m(z_2 - z_1) Y^\gamma(z_1; m) - \text{sgn}(z_2 - d) F'_m(z_2 - d) \Delta'_m. \quad (z_1 < d < z_2 \text{ or } z_1 > d > z_2)
\end{align*}
\]
The propagator matrices introduced here are the generalization of the Thomson–Haskell propagator matrix. Using the generalized propagator matrices we can compute the displacement and stress components at arbitrary depth in a unified way. The up-going and the down-going propagator matrices that have been used in previous studies can be regarded as a special case of the generalized propagator matrices.

2.4 Boundary conditions

In this subsection we describe the boundary conditions to be satisfied at the Earth’s surface and each layer interface in terms of the deformation matrices.

Continuity of displacement and stress continuity at each layer interface

At each layer interface the displacement components \((u_r, u_\varphi, u_z)\) and the stress components \((\sigma_z r, \sigma_z \varphi, \sigma_z z)\) must satisfy the following continuity conditions:

\[
\begin{align*}
\left. \begin{array}{l}
  u_i (r, \varphi, H_j+; j+1) = u_i (r, \varphi, H_j-; j) \\
  \sigma_{zi} (r, \varphi, H_j+; j+1) = \sigma_{zi} (r, \varphi, H_j-; j)
\end{array} \right\} (i = r, \varphi, z) \quad (31)
\end{align*}
\]

These boundary conditions are represented in terms of the deformation matrices as

\[
Y(H_j+; j+1) = D_j Y(H_j-; j), \quad Y'(H_j+; j+1) = D'_j Y'(H_j-; j),
\]

with

\[
D_j = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \mu_j/\mu_{j+1} & 0 \\
0 & 0 & 0 & \mu_j/\mu_{j+1}
\end{pmatrix}, \quad D'_j = \begin{pmatrix}
1 & 0 \\
0 & \mu_j/\mu_{j+1}
\end{pmatrix}.
\]

Stress-free conditions at the Earth’s surface

At the Earth’s surface the stress-free condition must be satisfied, including the gravitational effects associated with the surface uplift and subsidence (McConnell 1965; Matsu’ura & Sato 1989):

\[
\sigma_z r (r, \varphi, 0) = 0, \quad \sigma_z \varphi (r, \varphi, 0) = 0, \quad \sigma_z z (r, \varphi, 0) - \rho_1 g u_z (r, \varphi, 0) = 0 \quad (34)
\]

where \(\rho_1\) is the density of the surface layer and \(g\) is the acceleration due to gravity at the Earth’s surface. The third condition in eq. (34) can be written in the form of a wavenumber integral:

\[
\frac{\Delta u \mu_1}{4\pi} \int_0^\infty 2\xi^2 \mathbf{Y}_4(0; \xi; 1) \mathbf{J}(r, \varphi; \xi) d\xi = \rho_1 g \frac{\Delta u \mu_1}{4\pi} \int_0^\infty \xi \mathbf{Y}_2(0; \xi; 1) \mathbf{J}(r, \varphi; \xi) d\xi,
\]

where \(\mathbf{Y}_k\) represents the \(k\)th row of the total deformation matrix \(\mathbf{Y}\). Thus, the stress-free conditions at the Earth’s surface are represented in terms of the deformation matrices as

\[
\mathbf{Y}(0; 1) = \mathbf{G} \mathbf{Y}_0, \quad \mathbf{Y}'(0; 1) = \mathbf{Y}'_0,
\]

where

\[
\mathbf{Y}_0 = \begin{pmatrix}
\mathbf{Y}_0^0 \\
\mathbf{Y}_2^0 \\
0 \\
0
\end{pmatrix}, \quad \mathbf{Y}'_0 = \begin{pmatrix}
\mathbf{Y}_1^0 \\
0
\end{pmatrix}
\]

and

\[
\mathbf{G} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & -\rho_1 g/2\mu_1 & 0 & 1
\end{pmatrix}.
\]

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Finiteness of displacement and stress in the depths of the substratum

Since the displacement and stress components do not diverge at \( z \to \infty \) from causality, eq. (16) for the substratum \((j = n)\) is modified to

\[
Y^n(z; n) = E_n(z - H_{n-1})A_n, \quad Y^n(z; n) = E'_n(z - H_{n-1})A'_n,
\]

where

\[
A_n = \begin{pmatrix} a_n & -a_n \\ a_n & -a_n \end{pmatrix}, \quad A'_n = \begin{pmatrix} a'_n & -a'_n \\ a'_n & -a'_n \end{pmatrix},
\]

\[
E_n(z) = e^{-z z}, \quad E'_n(z) = e^{-z z},
\]

with

\[
a_n = \begin{pmatrix} A_{2n} & A_{1n} \\ B_{1n} & B_{2n} \end{pmatrix}, \quad a'_n = \begin{pmatrix} C_{1n} & C_{2n} \end{pmatrix}.
\]

2.5 Solution of the boundary-value problem

Using the generalized propagator matrices and the boundary conditions described in the preceding subsections, we can connect a series of deformation matrices from the top to the bottom (or from the bottom to the top) of the layered half-space. The matrix equations obtained can be solved for both of the surface displacements and the potential coefficients in the substratum but with different algorithms, as demonstrated in the following part of this subsection.

For the \(jh\) layer without any source \((j \neq m)\), using the down-going expressions for the generalized propagator matrices, we obtain the following relations from eqs (29) and (32):

\[
Y(H_j; j + 1) = D_j F_j(h_j) Y(H_{j-1}; j), \quad Y'(H_j; j + 1) = D'_j F'_j(h_j) Y'(H_{j-1}; j).
\]

For the \(nm\)th layer with a source at \( z = d \), from eqs (30) and (32) we obtain

\[
\begin{align*}
Y(H_m; m + 1) &= D_m F_m(h_m) Y(H_{m-1}; m) - D_m F_m(H_m - d) \Delta_m \\
Y'(H_m; m + 1) &= D'_m F'_m(h_m) Y'(H_{m-1}; m) - D'_m F'_m(H_m - d) \Delta'_m.
\end{align*}
\]

For the substratum, from eqs (28) and (39) we obtain

\[
\begin{align*}
Y(H_{n-1}; n) &= E_n(0) A_n + \delta_{nm} Y'(H_{n-1} - d; n) \\
Y'(H_{n-1}; n) &= A'_n + \delta_{nm} Y^n(H_{n-1} - d; n).
\end{align*}
\]

With these relations and the boundary conditions at the surface, we connect a series of deformation matrices from the top to the bottom of the layered half-space, and obtain the following matrix equations:

\[
A_n = P Y^n - Q^n, \quad A'_n = P' Y^n - Q'^n,
\]

with

\[
P = E_n^{-1}(0) \prod_{j=1}^{n-1} \begin{bmatrix} D_{n-j} F_{n-j}(h_{n-j}) \end{bmatrix} \begin{bmatrix} G \\ 0 \end{bmatrix}, \quad P' = \prod_{j=1}^{n-1} \begin{bmatrix} D'_{n-j} F'_{n-j}(h_{n-j}) \end{bmatrix}
\]

\[
Q^n = \begin{cases} 
E_n^{-1}(0) \prod_{j=1}^{n-1} \begin{bmatrix} D_{n-j} F_{n-j}(h_{n-j}) \end{bmatrix} D_m F_m(H_m - d) \Delta_m & (m \neq n) \\
E_n^{-1}(0) Y'(H_{n-1} - d; n) & (m = n)
\end{cases}
\]

\[
Q'^n = \begin{cases} 
E_n^{-1}(0) \prod_{j=1}^{n-1} \begin{bmatrix} D'_{n-j} F'_{n-j}(h_{n-j}) \end{bmatrix} D'_m F'_m(H_m - d) \Delta'_m & (m \neq n) \\
Y'(H_{n-1} - d; n) & (m = n)
\end{cases}
\]

Here, \(P\) and \(P'\) are the \(4 \times 4\) and \(2 \times 2\) matrices with the \(\xi\) dependence of \(exp\ (H_{n-1}\xi)\), and \(Q^n\) and \(Q'^n\) are the \(4 \times 3\) and \(2 \times 2\) matrices with the \(\xi\) dependence of \(exp\{(H_{n-1} - d)\xi\}\).
We can solve the matrix equations (46) for $Y^0$ and $Y^n$ (in practice for the surface displacements $Y^0_l$, $Y^0_1$ and $Y^n_l$) by eliminating the potential coefficients $A_k$ and $A'_k$ as

$$
\begin{align*}
Y_1^0 &= \frac{1}{\delta} \left( (P_{12} + P_{42}) (Q_1^n + Q_2^n) - (P_{12} + P_{32}) (Q_1^m + Q_2^m) \right), \\
Y_2^0 &= \frac{1}{\delta} \left( (P_{21} + P_{41}) (Q_1^n + Q_2^n) + (P_{21} + P_{41}) (Q_1^m + Q_2^m) \right),
\end{align*}
$$

(49)

with

$$
\delta = (P_{11} + P_{31})(P_{22} + P_{42}) - (P_{12} + P_{32})(P_{21} + P_{41}), \quad \delta' = P_{11} + P_{31},
$$

(50)

where $P_{ij}$ and $P'_{ij}$ are the ij elements of the matrices $P$ and $P'$, and $Q^n_m$ and $Q^m_n$ are the $k$th rows of the matrices $Q^n$ and $Q^m$. From the $\xi$ dependence of $P$, $P'$, $Q^n$ and $Q^m$ mentioned above, we can evaluate the $\xi$ dependence of $Y^0_1$, $Y^0_2$ and $Y^n_0$ as $\exp(-d\xi)$ at large $\xi$. As demonstrated by Sato (1971), Sato & Matsu'ura (1973) and Matsu'ura & Sato (1975), this guarantees numerical stability in computing surface deformation fields with the solution (49).

Substituting the solutions (49) into the original matrix equations (46), we can obtain the formal solutions for the potential coefficients in the substratum. In both solutions, however, both the first and second terms on the right-hand side of eq. (46) have the $\xi$ dependence of $\exp((H_{n-1} - d)\xi)$ at large $\xi$. When the source depth $d$ is smaller than $H_{n-1}$, this causes numerical instability, and so we cannot use the down-going algorithm to obtain the potential coefficients $A_k$ and $A'_k$.

Next, using the up-going expressions for the generalized propagator matrices, we connect a series of deformation matrices from the bottom to the top, and obtain the matrix equations parallel to eq. (46) in the following way. As noted in eq. (24), one of the most important properties of the generalized propagator matrices is reversibility. With this property we can rewrite eqs (43) and (44) as

$$
Y(H_{j-1}; j) = F_j(-\eta_j)D_j^{-1}Y(H_j; j + 1), \quad Y'(H_{j-1}; j) = F'_j(-\eta_j)D_j^{-1}Y'(H_j; j + 1) \quad (j \neq m)
$$

(51)

and

$$
\begin{align*}
Y(H_{m-1}; m) &= F_m(-\eta_m)D_m^{-1}Y(H_m; m + 1) + F_m(H_{n-1} - d)\Delta_m, \\
Y'(H_{m-1}; m) &= F'_m(-\eta_m)D_m^{-1}Y'(H_m; m + 1) + F'_m(H_{n-1} - d)\Delta'_m.
\end{align*}
$$

(52)

Connecting these relations with eqs (36) and (45), we obtain the following matrix equations:

$$
Y^0 = \tilde{P}A_0 + \tilde{Q}^m, \quad Y^n = \tilde{P}'A'_0 + \tilde{Q}'^m,
$$

(53)

with

$$
\tilde{P} = G^{-1} \prod_{j=1}^{n-1} \left[ F_j(-\eta_j)D_j^{-1} \right] E_0(0), \quad \tilde{P}' = \prod_{j=1}^{n-1} \left[ F'_j(-\eta_j)D_j^{-1} \right]
$$

(54)

$$
\begin{align*}
\tilde{Q}^m &= \left\{ \begin{array}{ll}
G^{-1} \prod_{j=1}^{n-1} \left[ F_j(-\eta_j)D_j^{-1} \right] F_m(H_{n-1} - d)\Delta_m & (m \neq n) \\
G^{-1} \prod_{j=1}^{n-1} \left[ F_j(-\eta_j)D_j^{-1} \right] Y'(H_{n-1} - d; n) & (m = n)
\end{array} \right.
\end{align*}
$$

(55)

$$
\begin{align*}
\tilde{Q}'^m &= \left\{ \begin{array}{ll}
\prod_{j=1}^{m-1} \left[ F'_j(-\eta_j)D_j^{-1} \right] F_m(H_{n-1} - d)\Delta'_m & (m \neq n) \\
\prod_{j=1}^{m-1} \left[ F'_j(-\eta_j)D_j^{-1} \right] Y'(H_{n-1} - d; n) & (m = n),
\end{array} \right.
\end{align*}
$$

where $\tilde{P}$ and $\tilde{P}'$ correspond to the inverse matrices of $P$ and $P'$ in eq. (47), respectively. Since the up-going propagator matrices $F_j(-\eta_j)$ and $F'_j(-\eta_j)$ include the exponential factor $\exp(k_j\xi)$, $\tilde{P}$ and $\tilde{P}'$ have the $\xi$ dependence of $\exp(H_{n-1}\xi)$, and $\tilde{Q}^m$ and $\tilde{Q}'^m$ have the $\xi$ dependence of $\exp(d\xi)$ for $m \neq n$ or $\exp((2H_{n-1} - d)\xi)$ for $m = n$. We can now solve the matrix equations (53) for the potential coefficients in the substratum as

$$
a_k = \frac{1}{\delta} \left( -(P_{42} - P_{43})Q_2^m + (P_{22} - P_{23})Q_1^m \right), \quad a'_k = -\frac{Q_2^m}{\delta'},
$$

(56)

with

$$
\delta = (P_{11} - P_{13})(P_{22} - P_{42}) - (P_{12} - P_{32})(P_{21} - P_{41}), \quad \delta' = P_{21} - P_{22},
$$

(57)

where $P_{ij}$ and $P'_{ij}$ are the ij elements of the 4 $\times$ 4 matrix $P$ and the 2 $\times$ 2 matrix $P'$, and $Q_1^m$ and $Q_2^m$ are the $k$th rows of the 4 $\times$ 3 matrix $Q^m$ and the 2 $\times$ 2 matrix $Q'^m$. The solutions $a_k$ and $a'_k$ in eq. (56) are numerically stable, because they have the $\xi$ dependence of $\exp(-|H_{n-1} - d|\xi)$ at large $\xi$. Substituting these solutions into the original matrix equations (53), we can obtain the formal solutions of the surface displacements $Y^0_1$, $Y^0_2$ and $Y^n_0$, but with the $\xi$ dependence of $\exp(d\xi)$ for $m \neq n$ or $\exp((2H_{n-1} - d)\xi)$ for $m = n$ at large $\xi$. This means that numerical stability in computing the surface deformation fields is not guaranteed, if we use the up-going algorithm like Singh (1970), Jovanovich et al. (1974a) and Rundle (1978).

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2.6 Internal deformation fields due to a point dislocation source

In the preceding subsection we derived numerically stable solutions for both the deformation matrices, \( Y^0 \) and \( Y^0 \), at the surface, and the potential coefficient matrices, \( A_n \) and \( A'_n \), in the substratum in a unified way. From these solutions, using the generalized propagator matrices, we can obtain the deformation matrices \( Y(z; j) \) and \( Y'(z; j) \) at arbitrary depth in the layered elastic half-space as follows.

At any depth in the substratum \( z \geq H_{n-1} \), regardless of the source depth, the expressions for the deformation matrices are given by

\[
Y(z; n) = E_n(z - H_{n-1})A_n + \delta_{n}\gamma Y'(z - d; n), \quad Y'(z; n) = E'_n(z - H_{n-1})A'_n + \delta_{n}\gamma Y''(z - d; n).
\]  

(58)

In the other cases, at any depth shallower than the source \( 0 < z < d \), the deformation matrices can be obtained by using the down-going expressions for the generalized propagator matrices, as

\[
Y(z; j) = F(z - H_{j-1})D^{-1}_{j} \prod_{l=j+1}^{\infty} [F(l-h_j)D^{-1}_{j}] \gamma Y^0, \quad Y'(z; j) = F(z - H_j)D^{-1}_{j} \prod_{l=j+1}^{\infty} [F(l-h_j)D^{-1}_{j}] \gamma Y^0.
\]  

(59)

and at any depth deeper than the source \( d < z < H_{n-1} \), by using the up-going expressions, as

\[
Y(z; j) = F(z - H_j)D^{-1}_{j} \prod_{l=j+1}^{\infty} [F(l-h_j)D^{-1}_{j}] \gamma Y^0, \quad Y'(z; j) = F(z - H_j)D^{-1}_{j} \prod_{l=j+1}^{\infty} [F(l-h_j)D^{-1}_{j}] \gamma Y^0.
\]  

(60)

The deformation matrices defined in eqs (58)-(60) have the \( \xi \) dependence of \( \exp(-|z - d|\xi) \) at large \( \xi \). Using these expressions for the deformation matrices, we can compute the internal displacement and stress fields at any depth in the layered elastic half-space without numerical instability.

2.7 Expressions for a line dislocation source

In order to obtain the deformation fields caused by a finite-dimensional fault, we must numerically integrate the solutions for point dislocation sources over a fault surface. This procedure is usually very time consuming. In some problems, such as the crustal deformation due to steady plate subduction, however, our interest is not the spatial variation of deformation fields along the plate boundary, but is instead the vertical section of deformation fields across the plate boundary. In such cases the original 3-D problem can be treated as a 2-D problem for approximation. Thus, it is useful to provide the expressions for the deformation fields for a line dislocation source here.

We consider a line dislocation source parallel to the \( x \)-axis, located at \( y = 0 \) and \( z = d \) and with a dip angle \( \theta \) and a slip angle \( \chi \) (Fig. 1). To obtain the displacement and stress fields due to the line source, we first transform the expressions for point-source solutions in cylindrical approximation. Thus, it is useful to provide the expressions for the deformation fields for a line dislocation source here.

On the basis of the mathematical expressions derived in the preceding section, we can code a computer program to calculate the internal displacement and stress fields due to faulting without difficulty. Since the problem of numerical instability has already been solved, we do not

3 NUMERICAL EXAMPLES

On the basis of the mathematical expressions derived in the preceding section, we can code a computer program to calculate the internal displacement and stress fields due to faulting without difficulty. Since the problem of numerical instability has already been solved, we do not
need to pay special attention to the numerical evaluation of the semi-infinite integrals with respect to a wavenumber. In this section we give some examples of numerical computation.

First, we consider the case of a rectangular fault embedded in a two-layer elastic half-space (Fig. 2), where the fault plane extends from \( x = -100 \) to \( 100 \) km and from \( z = 15 \) to \( 50 \) km with a dip angle of \( 30^\circ \). The slip angle and the fault offset are taken to be \( 45^\circ \) and \( 1 \) m, respectively. The values of the structural parameters used for computation are given in Table 1. The computed displacement field is shown in Fig. 3. The thick solid lines in Fig. 3(a) represent the profiles of vertical displacements along the \( y \)-axis \( (x = 0) \) at \( z = 20, 40 \) and \( 60 \) km. The dotted and broken lines represent the vertical displacement profiles at \( x = -80 \) and \( 80 \) km, respectively. In Fig. 3(a) the vertical section of the rectangular fault is also shown by the solid line, with the arrows indicating the direction of fault slip. The series of vector maps in Fig. 3(b) represents the horizontal displacement fields at \( z = 0, 20, 40 \) and \( 60 \) km. Here, the rectangular region enclosed by the broken line indicates the horizontal projection of the fault plane, and the thick solid line indicates the horizontal section of the fault plane at each depth. From Fig. 3 we can see that the effects of the free surface on the internal displacement fields weaken rapidly with depth. For example, in the shallow region, the displacements on the hangingwall side are much larger than those on the footwall side. Such asymmetric patterns of the displacement field cannot be found in the deep region.

Next, we consider the case in which the rectangular fault in Fig. 2 extends infinitely in the direction parallel to the fault strike, as indicated by the two parallel broken lines. All the other model parameters, the structural parameters and the fault parameters except for the fault length, are taken to be the same as those in the case of Fig. 3. In Fig. 4 we show the internal displacement fields due to the infinitely long rectangular fault embedded in the two-layer elastic half-space. The thick solid lines in Fig. 4(a) represent the profiles of vertical displacements at \( x = 0, 20, 40 \) and \( 60 \) km. The solid arrows in Fig. 4(b) represent the horizontal displacement fields at \( z = 0, 20, 40 \) and \( 60 \) km. The displacement patterns in Fig. 4 do not change in the direction parallel to the fault strike. The displacement fields shown in Fig. 4 are nearly the same as those along the \( y \)-axis \( (x = 0) \) in Fig. 3. This means that the line-source approximation is valid for a sufficiently long fault (e.g. \( >200 \) km).

In Fig. 4, to see the effects of layering, we also show the displacement fields due to the same infinitely long rectangular fault in an elastic half-space. Here, the vertical and horizontal displacement fields are indicated by the broken lines in (a) and by the open arrows in (b), respectively. In this computation, we suppose the same elastic property of the half-space as that of the first layer of the two-layer model in Table 1, and neglect the effect of gravity. In the case of a non-gravitating homogeneous elastic half-space, which has commonly been assumed in the computation of coseismic crustal deformation, the analytic expressions for internal deformation fields due to faulting have been obtained by many researchers (e.g. Iwasaki & Sato 1979; Okada 1992). From comparison of the two displacement fields in Fig. 4, we can see that the discrepancy between the half-space model and the two-layer model is as much as \( 10-20 \) per cent. The effect of gravity on the elastic displacement fields is almost negligible (Iwasaki & Matsu’ura 1982). If we take a larger contrast in rigidity between the surface layer and the elastic substratum, the discrepancy becomes more significant (e.g. Sato & Matsu’ura 1973; Jovanovich et al. 1974b).

### 4 Discussion and Conclusions

In Section 2 we defined the generalized propagator matrix, which includes both the up-going propagator matrix defined by Singh (1970) and the down-going propagator matrix defined by Sato (1971) as special cases. With the generalized propagator matrices we succeeded in

---

**Table 1.** Two-layer structure model used for computation.

<table>
<thead>
<tr>
<th>No.</th>
<th>( H ) (km)</th>
<th>( V_F ) (km s(^{-1}))</th>
<th>( V_S ) (km s(^{-1}))</th>
<th>( \rho ) (kg m(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>6.0</td>
<td>3.5</td>
<td>( 2.6 \times 10^3 )</td>
</tr>
<tr>
<td>2</td>
<td>( \infty )</td>
<td>8.0</td>
<td>4.5</td>
<td>( 3.4 \times 10^3 )</td>
</tr>
</tbody>
</table>

---

**Figure 2.** Geometry of the rectangular fault embedded in a two-layer elastic half-space. The structural parameters of the two-layer model are given in Table 1. The rectangular fault extends from \( x = -100 \) to \( 100 \) km and from \( z = 15 \) to \( 50 \) km. The dip angle, the slip angle and the fault offset are taken to be \( 30^\circ, 45^\circ \) and \( 1 \) m, respectively. The two parallel thick broken lines indicate the rectangular fault extending infinitely in the direction parallel to the \( x \)-axis.
Figure 3. Internal displacement fields due to a rectangular fault in the two-layer elastic half-space. The geometry of the fault is shown in Fig. 2. (a) Vertical displacements at $z = 0, 20, 40$ and $60$ km. The solid, dotted, and broken lines represent the vertical displacement profiles at $x = 0, -80$ and $80$ km, respectively. The solid line with the arrows indicates the vertical section of the fault plane. (b) Horizontal displacements at $z = 0, 20, 40$ and $60$ km. In each diagram the rectangular region enclosed by the broken lines indicates the horizontal projection of the fault plane, and the thick solid line indicates the horizontal section of the fault plane.
Figure 4. Internal displacement fields due to an infinitely long rectangular fault in the two-layer elastic half-space. The geometry of the fault is shown in Fig. 2 by the two parallel thick broken lines. (a) Vertical displacements at $z = 0, 20, 40$ and 60 km (thick solid lines). The solid line with the arrows indicates the vertical section of the fault plane. (b) Horizontal displacements at $z = 0, 20, 40$ and 60 km (solid arrows). In each diagram the broken lines indicate the horizontal extent of the fault plane, and the thick solid line indicates the horizontal section of the fault plane. The vertical and horizontal displacement fields due to the same fault model in an elastic half-space are also shown by the broken lines in (a) and by the open arrows in (b), respectively. The elastic property of the half-space is taken to be the same as that of the first layer of the two-layer model in Table 1.
obtaining general expressions for the internal deformation fields due to a point dislocation source in a gravitating layered elastic half-space in a unified way. Through a stability check of the derived solutions we demonstrated that (1) the solution obtained with the up-going algorithm is stable below the source, but becomes unstable above the source, and (2) the solution obtained with the down-going algorithm is stable above the source, but becomes unstable below the source.

The solution of surface displacements derived by Sato (1971) has been cited as an example of numerically unstable solutions (e.g. Jovanovich et al. 1974a; Wang et al. 2003), but Sato’s solution obtained with the down-going algorithm was actually stable at the surface. As stated in Sato (1971), the oscillations of kernel functions in figs 3–5 of his paper do not cause numerical instability, because these kernel functions are multiplied by a function exponentially decreasing with wavenumber.

The mathematical formulation by Singh (1970) and that by Sato (1971) seem to be very similar to each other, but there are some essential differences. One major difference is in the source representation. A further, most essential, difference, which has not been pointed out, is in the definition of propagator matrices: Singh (1970) used the up-going propagator matrix, while Sato (1971) used the down-going propagator matrix. Thus, Singh’s solution obtained with the up-going algorithm was numerically unstable at the surface.

In Section 2 we also gave the solution for the internal displacement and stress fields due to a line dislocation source. For the line dislocation source, Singh & Grag (1985) and Sato & Matsu’ura (1993) obtained expressions for surface displacement by using the up-going and the down-going propagator matrices, respectively.

As demonstrated by the numerical examples, the difference between the non-gravitating elastic half-space model and the gravitating two-layer elastic half-space model is not so significant for the internal displacement fields. As pointed out by many researchers (e.g. Thatcher & Rundle 1984; Matsu’ura & Sato 1989; Fukahata et al. 2004), however, the difference between these two models becomes crucial in problems of long-term (>10 yr) crustal deformation, where we cannot neglect the viscoelastic property of the asthenosphere. Applying the correspondence principle of linear viscoelasticity (Lee 1955; Radok 1957) to the elastic solutions derived in this study, we can obtain the internal viscoelastic deformation fields in a multilayered half-space. In a subsequent paper, we shall give the solution for the internal viscoelastic deformation fields due to a dislocation source.

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