Short Note
Omori-Utsu Law c-Values Associated with Recent Moderate Earthquakes in Japan
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Abstract We investigate the early aftershock activity associated with four moderate earthquakes ($M_w$ 6.6–6.7) that occurred recently in Japan. For each aftershock sequence, we examine continuous high-pass filtered seismograms recorded at seismic stations nearby the main fault to identify as many early events as possible. The magnitude of these events is calibrated using aftershocks that are listed in the earthquake catalog of Japan Meteorological Agency (JMA). The analysis of the aftershock decay rates reveals a power-law time dependence with a scaling exponent close to 1.0 that starts from about one minute from the mainshock. Our results demonstrate that the $c$-value of the Omori–Utsu law is very small, although a lower bound is not established due to completeness problems in the first minute after the mainshock and statistical fluctuations.

Introduction

Large shallow earthquakes are followed by intense aftershock activity that decays with time approximately as a power law (Omori, 1894). The occurrence rate of aftershocks is usually described well by the Omori–Utsu law (Utsu, 1961),

$$n(t) = \frac{k}{(t + c)^p},$$

(1)

where $n(t)$ is the frequency of aftershocks per unit time at time $t$ after the main shock, and $k$, $c$, and $p$ are constants. The exponent $p$ indicates how fast the aftershock rate decays with time and has a value close to 1.0 (Utsu et al., 1995). The parameter $k$ corresponds to the aftershock productivity and $c$-value is a constant time shift that relates to the rate of aftershocks in the early part of an aftershock sequence and typically ranges from 0.5 to 20 hr in empirical studies (Utsu et al., 1995).

The existence of $c$-value and its physical meaning are still under debate (e.g., Utsu et al., 1995; Enescu and Ito, 2002; Kagan, 2004; Shcherbakov et al., 2004). The large amplitudes of the coda waves of the mainshock as well as the large number of aftershocks that occur in a relatively short time interval, hinder the detection and localization of small, early aftershocks (e.g., Kagan, 2004; Peng et al., 2007; Kilb et al., 2007). However, with the improved recording capabilities of seismic networks, one can get a more accurate image of the early part of an aftershock sequence, which holds valuable information about the underlying mechanisms that control the occurrence of aftershocks (e.g., Dieterich, 1994; Gomberg, 2001; Rubin, 2002). Thus, recent analyses of high-frequency continuous waveforms at stations situated closely to the mainshock source region revealed important characteristics of aftershock activity in the first few minutes after a large earthquake (Peng et al., 2006; Enescu et al., 2007; Mori et al., 2008). The $c$-values reported in these studies are very small, that is, less than a few minutes, although it is not clear which physical mechanism is most appropriate to explain the results. On the other hand, other recent studies (Shcherbakov et al., 2004; Shcherbakov and Turcotte, 2006; Nanjo et al., 2007) proposed much larger $c$-values based on the analysis of earthquake catalog data.

In this study we systematically analyze the early aftershock activity following four moderate crustal earthquakes in Japan. We use continuous waveform data, recorded mostly by the high sensitivity seismic network (Hi-net) in Japan, as well as JMA catalog data. We carefully tackle the early catalog incompleteness, provide reliable $c$-value estimations for these sequences, and discuss physical and earthquake hazard related implications of our results.

Data and Analysis Procedure

We analyze the aftershock sequences of four moderate recent Japanese earthquakes: (1) the 2000 western Tottori...
(M 7.3 and Mw 6.6), (2) the 2005 Fukuoka (M 7.0 and Mw 6.6), (3) the 2007 Noto peninsula (M 6.9 and Mw 6.7), and (4) the 2007 Chuetsu–Oki (M 6.8 and Mw 6.6), where M and Mw are the JMA magnitude and the moment magnitude, respectively.

For each sequence, we consider the events in the JMA catalog (see the Data and Resources section) that occur in a time window of 90 days from the mainshock. For consistency the same time window is used for all the analyzed sequences. We also experimented with shorter or longer time windows and found no significant change of the results. Because of the relatively low background seismicity in the four regions, we could easily delineate the spatial distributions of the aftershocks that generally cluster on and around the faults of the studied areas. We also considered the case of a spatial window that scales with the magnitude of each mainshock (e.g., Kagan, 2002) and obtained similar results. Figures 1a–d show the epicentral distributions of aftershocks recorded by JMA and selected for analysis.

While the catalog data of JMA are very good, and the magnitude of completeness (Mc) is usually less than 2.0 over major parts of Japan, there are a significant number of aftershocks that occur in the first few minutes after a large mainshock that are not recorded in the JMA catalog. To detect as many early aftershocks as possible we use continuous waveform data recorded at Hi-net and the Earthquake Research Institute (ERI), University of Tokyo, seismic stations (see the Data and Resources section and Okada et al., 2004) located within 55 km of the aftershocks distributions (Figs. 1a–d). Each station consists of a three-component velocity seismometer with a sampling rate of 100/sec. For each sequence, we process vertical and horizontal component continuous waveforms recorded at six seismic stations. The examination of seismograms for early aftershock detection was carried out for a time period of 25 min in the case of western Tottori, Fukuoka, and Chuetsu–Oki earthquakes and 12 min in the case of Noto earthquake due to the limited availability of continuous waveform data for this sequence.

Figure 1. Epicentral maps of the (a) 2000 western Tottori, (b) 2005 Fukuoka, (c) 2007 Noto, and (d) 2007 Chuetsu–Oki aftershock sequences (M ≥ 2.0). A star marks the epicenter of the mainshock. The seismic stations used for the analysis of continuous waveform data are shown as rectangles and their names are also indicated. Most stations belong to the Hi-net network except E.AKIS, E.ATRG, and E.WGM stations in the Noto region, which are operated by ERI.
The procedure used to detect early aftershocks on the seismograms and estimate their magnitudes is similar to that applied by Enescu et al. (2007) and Mori et al. (2008). Because the coda waves of the mainshock have little high-frequency content, we apply a Butterworth 5 Hz high-pass filter to detect early aftershocks hidden by the mainshock coda. We have also tried several higher cut-off frequencies of up to 25 Hz and obtain similar results. In a few cases smaller magnitude early events are easier to identify on seismograms filtered with higher frequency, however, the high-frequency noise is also amplified in such a case, and this hinders the correct picking of events. We analyze first the frequency content, we apply a Butterworth 5 Hz high-pass filter to detect early aftershocks hidden by the mainshock coda. We have also tried several higher cut-off frequencies (Table 1). Similar results are obtained if one uses the first 100 events of each sequence to estimate the b-value, however, the statistics are less reliable. Note that the estimated $M_c$ values are only approximate measures of the completeness level for our data sets, and they might be significantly larger in the first minute or so after the mainshock, where our seismogram picking is probably less complete (see also Fig. 2). In the following analysis of aftershock decay we adopted a magnitude threshold of 3.5 for all sequences to make the results comparable and also to be above the completeness levels estimated from the frequency–magnitude distributions and the examination of signal-to-noise ratio of seismograms.

Figure 2 and especially Figure 3 give some indication on the completeness magnitude of our data. We further present in Figure 4 the frequency–magnitude distribution for the first 200 events of each aftershock sequence that occurred within 1–2 hr from their corresponding mainshock. Following a similar procedure to that by Wiemer and Wyss (2000), we determined $M_c$ as the magnitude at which 95% of the data can be modeled by a power-law fit. The $b$-value in each case (Table 1) is obtained using a maximum likelihood procedure (Aki, 1965; Utsu, 1965), considering only the earthquakes with $M \geq M_c$. $M_c$ is 3.1 for Tottori and 3.3 for the other three aftershock sequences, while $b$-value is close to one in all cases (Table 1). Similar results are obtained if one uses the first 100 events of each sequence to estimate the $b$-value, however, the statistics are less reliable. Note that the estimated $M_c$ values are only approximate measures of the completeness level for our data sets, and they might be significantly larger in the first minute or so after the mainshock, where our seismogram picking is probably less complete (see also Fig. 2).

**Results**

Figure 2 shows examples of high-pass filtered continuous seismograms recorded at four seismic stations situated closely to the epicentral areas of the 2000 western Tottori, the 2005 Fukuoka, the 2007 Noto, and the 2007 Chuetsu–Oki aftershock sequences, respectively. The time origin corresponds to the P-wave arrival of the mainshock at the closest seismic station. The arrival times identified at the closest stations were used to search for the $P$ and $S$ phases of the same events on seismograms recorded at the other seismic stations. To determine the magnitude of an event, we measured the corresponding peak waveform amplitudes at the six nearby stations. The magnitude was calibrated using a set of 30 earthquakes that were also listed in the catalog of JMA. The arrival times identified at the closest stations were used to search for the $P$ and $S$ phases of the same events on seismograms recorded at the other seismic stations. To determine the magnitude of an event, we measured the corresponding peak waveform amplitudes at the six nearby stations. The magnitude was calibrated using a set of 30 earthquakes that were also listed in the catalog of JMA.

The maximum likelihood values of the $c$ parameter for the four analyzed sequences (Figs. 5a–d and Table 1) are on the order of one minute. If one considers the standard deviation of the parameter, the upper limit of the $c$-value is of 0.6 min, 2.4 min, 1.17 min, and 1.12 min for the combined catalogs of the Tottori, Fukuoka, Noto, and Chuetsu–Oki sequences, respectively. Note that the real (i.e., unbiased by data incompleteness) $c$-values for these sequences might be even smaller due to the difficulty of detecting early aftershocks within the first $\sim 90$ sec from the mainshock (Figs. 2 and 3). As can be noticed in Figures 2 and 5, the aftershocks

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Figure 2. Velocity seismograms high-pass filtered at 5 Hz for the early portions of the 2000 western Tottori, the 2005 Fukuoka, the 2007 Noto, and the 2007 Chuetsu–Oki aftershock sequences. The recording stations are N.HKTH, N.GENH, E.AKIS, and N.NGOH, respectively. The horizontal gray lines indicate the approximate amplitude levels for $M_{3.5}$ earthquakes. The velocity seismograms were scaled so the amplitude corresponding to an $M_{3.5}$ earthquake is the same for the four cases.
of the 2000 Tottori earthquake are sharper and go back to earlier times than the aftershocks of the other three mainshocks. This is likely caused by the station distribution, that is, the nearest recording station is closer to the aftershock zone for the 2000 Tottori earthquake, so more aftershocks are identified in this case. It is also interesting to note that the aftershock sequence with the largest epicentral distance to the nearest seismic station (i.e., Fukuoka sequence) has the largest $c$-value. We also performed the same aftershock decay analysis for threshold magnitudes ranging from 2.5 to 3.4. The estimated $c$-values are slightly larger but remain on the order of one minute if the cut-off magnitudes are equal to or larger than the completeness thresholds determined for each sequence (Fig. 4) with the exception of the Fukuoka sequence for which the $c$-value equals 2.6 min for a cut-off magnitude of 3.3. At a threshold magnitude of 2.5, the smallest $c$-value of 4.3 min is obtained for the Tottori sequence, which has the closest nearby station and the best azimuthal coverage with seismic stations among the four studied sequences.

The original JMA aftershock data are characterized by significantly larger $c$-values, which obviously reflect the catalog incompleteness immediately after the mainshock.

**Discussion**

The results presented in the previous section strongly argue for a very small $c$-value of about one minute or less. The good quality of waveform data made possible to accurately pick and determine the magnitude of aftershocks that occur after about 60–90 sec from the mainshock. These events, especially those with larger magnitudes, could be clearly identified at several seismic stations, so the accuracy of magnitude determination is high. The picking and magnitude determination becomes less accurate for times from the mainshock of less than one minute, which makes it difficult to physically interpret $c$-values of the same order. Peng et al. (2007) identified early aftershocks that occur from about 20 sec after the corresponding mainshocks using high-pass filtered Hi-net seismograms. In their case, the magnitudes of the studied mainshocks range from 3 to 5. Because the coda wave for such smaller events is shorter than for the earthquakes analyzed here, their picking accuracy at very early times might have been higher.

The 2000 Tottori and 2005 Fukuoka earthquakes were strike-slip events, while the 2007 Noto and 2007 Chuetsu–Oki earthquakes had a thrust-type focal mechanism. Thus,
our results suggest that aftershock sequences that occur in different tectonic environments are characterized by a similarly small $c$-value. Peng et al. (2006) observed a steady rate of aftershocks in the first 2.2 min after the 2004 $M_w 6.0$ Parkfield earthquake followed by a power-law decay of aftershock activity. Enescu et al. (2007) determined a $c$-value of about 4.3 min for the 2004 $M_w 6.6$ Chuetsu aftershock sequence and interpreted the result using a rate-and-state model (Dieterich, 1994). This sequence, however, had several large early aftershocks, and if one accounts for the time-decay of the secondary aftershocks, the $c$-value might be smaller (Enescu et al., 2007; Mori et al., 2008). As argued by Mori et al. (2008) the abundance of large aftershocks for the 2004 Chuetsu earthquake could be an effect of enhanced fluid induced triggering. All these results strongly support the idea (Kagan, 2004) that $c$-values larger than a few minutes are an artifact of catalog incompleteness in the early stages of an aftershock sequence.

Shcherbakov et al. (2004) and Shcherbakov and Turcotte (2006), using aftershock data from southern California, proposed that the $c$-value of the Omori–Utsu law is not a constant but scales with a lower magnitude cutoff of the aftershocks. They suggest that the $c$-value plays the role of a characteristic time for the establishment of the Gutenberg–

![Frequency–magnitude distribution for the first 200 events in each combined catalog](image)

**Figure 4.** Frequency–magnitude distribution for the first 200 events in each combined catalog: (a) the 2000 western Tottori, (b) the 2005 Fukuoka, (c) the 2007 Noto, and (d) the 2007 Chuetsu–Oki sequence, respectively.

<table>
<thead>
<tr>
<th>Aftershock Sequence</th>
<th>$p$-Value</th>
<th>$k$-Value</th>
<th>$c$-Value (sec)</th>
<th>$a$-Value</th>
<th>$b$-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000 Western Tottori ($M_w 6.6$)</td>
<td>1.05</td>
<td>8.07</td>
<td>16.4</td>
<td>5.05</td>
<td>0.99</td>
</tr>
<tr>
<td>2005 Fukuoka ($M_w 6.6$)</td>
<td>0.99</td>
<td>10.32</td>
<td>62.2</td>
<td>5.27</td>
<td>1.07</td>
</tr>
<tr>
<td>2007 Noto ($M_w 6.7$)</td>
<td>1.06</td>
<td>11.64</td>
<td>31.1</td>
<td>5.07</td>
<td>0.94</td>
</tr>
<tr>
<td>2007 Chuetsu–Oki ($M_w 6.6$)</td>
<td>1.15</td>
<td>4.01</td>
<td>14.7</td>
<td>4.46</td>
<td>0.84</td>
</tr>
</tbody>
</table>
Richter scaling. Nanjo et al. (2007) used Japanese aftershock data to reach similar conclusions. All the c-values reported in these studies for aftershock data with a threshold magnitude of 3.5 are, however, significantly larger than those found in this study with most of them being larger than about 20 min. Our results suggest that the physical scaling proposed in these papers is likely an effect of incomplete aftershock data sets.

The rate-and-state model (Dieterich, 1994) is widely used to explain the decay of aftershock activity. Knowledge of aftershocks rate, especially for the early part of a sequence, can be used to estimate the stress heterogeneity on the mainshock rupture area (Helmstetter and Shaw, 2006; Marsan, 2006). Thus, a small c-value could be a consequence of a heterogeneous Coulomb stress change with relatively large peak values (Dieterich, 1994; Helmstetter and Shaw, 2006). The decay rate of early aftershocks is very important in aftershock hazard studies (Woessner et al., 2008). Therefore, our findings are important to constrain the physical parameters that control the aftershocks occurrence process and provide useful information for aftershock forecasting studies.

Conclusions

We have analyzed high-frequency waveforms recorded at Hi-net seismic stations to investigate the early aftershock activity of four moderate recent Japanese earthquakes (Mw 6.6–6.7). We found that a significant fraction of aftershocks in the first minutes after the mainshocks is missing from the JMA earthquake catalog. Careful event-picking and magnitude determination of early aftershocks detected on seismograms enabled us to obtain more complete aftershock data sets.

Our results indicate that a power law with a scaling exponent p close to 1.0 describes well the decay of aftershock activity from times of about one minute from the mainshock. It is, however, difficult to accurately determine the aftershocks decay in the first 60–90 sec after the mainshock due...
to completeness problems and statistical fluctuations. The observation of very small c-values for aftershock sequences that occur in regions with different tectonic characteristics strongly suggests that this may be a general feature of aftershocks decay and, therefore, commonly reported c-values larger than a few minutes are caused by missing aftershocks in the earthquake catalog. Our results have important consequences for the models used to explain the decay of aftershocks and provide useful information for aftershock forecasting studies.

Data and Resources

The continuous waveforms used in this study were obtained from the opened data archive of Hi-net (www.hinet.bosai.go.jp, last accessed September 2008) maintained by NIED. The E.ATRG and E.AKIS stations data were collected as part of the project, The Japanese University Joint Seismic Observations at the Niigata-Kobe Tectonic Zone (NKTZ). JMA catalog data is also open to the public and freely provided to the Japanese Universities and Institutions that cooperate with JMA.

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